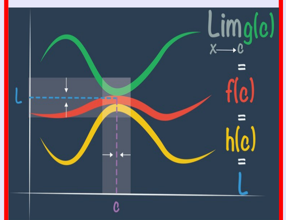
Math 261

Spring 2021

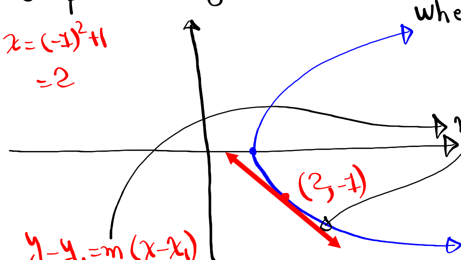
Lecture 20



Graph $x = y^2+1$

Label the point where y is -1.

$x = (-1)^2+1$
$= 2$



$m = \frac{dy}{dx}\Big|_{(2,-1)}$

$y - y_1 = m(x - x_1)$

$y-(-1) = \frac{-1}{2}(x-2)$

$y+1 = -\frac{1}{2}x + 1$

$\boxed{y = -\frac{1}{2}x}$

Find equation of this tan. line.

$x = y^2+1$

$\frac{d}{dx}[x] = \frac{d}{dx}[y^2+1]$

$1 = \frac{d}{dx}[y^2] + \frac{d}{dx}[1]$

$1 = 2y \cdot \frac{dy}{dx} + 0$

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

Implicit Differentiation

$m = \frac{dy}{dx}\Big|_{(2,-1)} = \frac{1}{2(-1)} = \boxed{\frac{-1}{2}}$

Consider $x^2+y^2=100$, find eqn of the tan. line at the point $(6,8)$

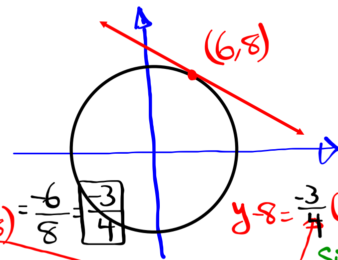
Circle
center $(0,0)$
radius 10

Verify that $(6,8)$ is on the Curve of $x^2+y^2=100$.

$6^2+8^2=100$

$36+64=100$

$100=100$ ✓



$m=\frac{dy}{dx}\Big|_{(6,8)}=\frac{-6}{8}=\boxed{\frac{-3}{4}}$

$y-8=\frac{-3}{4}(x-6)$

Simplify

$\boxed{y=\qquad}$

$x^2+y^2=100$

$\frac{d}{dx}\left[x^2+y^2\right]=\frac{d}{dx}[100]$

$2x+2y\cdot\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$y\frac{dy}{dx}=-x$

$\frac{dy}{dx}=\frac{-x}{y}$

Find eqn of the tangent line to the graph of

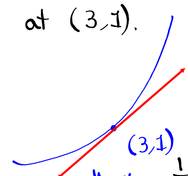
$x^2y+6=5xy^2$ at $(3,1)$.

Verify the point:

$3^2\cdot 1+6=5\cdot 3\cdot 1^2$

$9+6=15$

$15=15$ ✓



$y-1=\frac{1}{29}(x-3)$

Simplify

$m=\frac{dy}{dx}\Big|_{(3,1)}$

$\boxed{y=\qquad}$

$x^2y+6=5xy^2$

$\frac{d}{dx}\left[x^2y+6\right]=\frac{d}{dx}\left[5xy^2\right]$

$\frac{d}{dx}\left[x^2y\right]+\cancel{\frac{d}{dx}[6]}^{0}=5\frac{d}{dx}\left[xy^2\right]$

$2x\cdot y+x^2\frac{dy}{dx}=5\left[1\cdot y^2+x\cdot 2y\cdot\boxed{\frac{dy}{dx}}\right]$

$2xy+x^2\frac{dy}{dx}=5y^2+10xy\frac{dy}{dx}$

$x^2\frac{dy}{dx}-10xy\frac{dy}{dx}=5y^2-2xy$

$(x^2-10xy)\frac{dy}{dx}=5y^2-2xy$

$\frac{dy}{dx}=\frac{5y^2-2xy}{x^2-10xy}$

$m=\frac{dy}{dx}\Big|_{(3,1)}=\frac{5(1)^2-2(3)(1)}{1^2-10(3)(1)}=\frac{5-6}{1-30}=\frac{-1}{-29}=\boxed{\frac{1}{29}}$

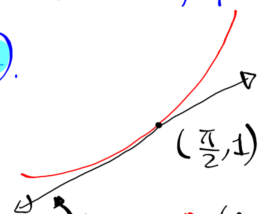
Find eqn of the tan. line to the graph of

$\sin xy = y$ at $\left(\frac{\pi}{2}, 1\right)$.

$\sin \frac{\pi}{2} \cdot 1 = 1$

$\sin \frac{\pi}{2} = 1$

$1 = 1 \checkmark$



$y - 1 = 0\left(x - \frac{\pi}{2}\right)$

Final Ans. $\boxed{y = 1}$

$\frac{d}{dx}\left[\sin xy\right] = \frac{d}{dx}\left[y\right]$

$\cos xy \cdot \left[1 \cdot y + x\frac{dy}{dx}\right] = \frac{dy}{dx}$

$y\cos xy + x\cos xy\frac{dy}{dx} = \frac{dy}{dx}$

$x\cos xy\frac{dy}{dx} - \frac{dy}{dx} = -y\cos xy$

$(x\cos xy - 1)\frac{dy}{dx} = -y\cos xy$

$\frac{dy}{dx} = \frac{-y\cos xy}{x\cos xy - 1}$

$m = \left.\frac{dy}{dx}\right|_{\left(\frac{\pi}{2}, 1\right)} = \frac{-1\cancel{\cos\frac{\pi}{2}}^{0} \cdot 1}{\frac{\pi}{2}\cancel{\cos\frac{\pi}{2}}^{0} \cdot 1 - 1} = \frac{0}{-1} = 0$

$\boxed{m = 0}$

Given $x^2 - 4x + y^2 + 3 = 0$

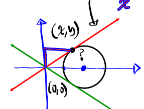
Can you graph this eqn?

$\underbrace{x^2 - 4x + 4} + y^2 + 3 = 4$

$(x-2)^2 + (y-0)^2 = 1$

Circle Center $(2,0)$, $r = 1$

$m = \frac{y}{x}$



Find $\boxed{\text{two tan. lines}}$ that contain the origin to this graph.

$\frac{d}{dx}\left[x^2 - 4x + y^2 + 3\right] = \frac{d}{dx}[0]$

$2x - 4 + 2y\frac{dy}{dx} + 0 = 0$

$2y\frac{dy}{dx} = 4 - 2x$

$\frac{dy}{dx} = \frac{4-2x}{2y}$

$\frac{dy}{dx} = \frac{2-x}{y}$

$\frac{2-x}{y} = \frac{y}{x}$

Curve line

Cross-Multiply

$y \cdot y = x(2-x)$

$y^2 = 2x - x^2$

$x^2 + y^2 - 2x = 0$

$-\begin{cases} x^2 - 4x + y^2 + 3 = 0 \\ x^2 + y^2 - 2x = 0 \end{cases}$ Curve

slopes = deri.

$2x - 3 = 0$ $\quad x = \frac{3}{2}$

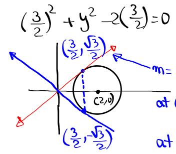
$x^2 + y^2 - 2x = 0$

$\left(\frac{3}{2}\right)^2 + y^2 - 2\left(\frac{3}{2}\right) = 0$

$\frac{9}{4} + y^2 - 3 = 0$

$y^2 = 3 - \frac{9}{4}$ $\quad y^2 = \frac{3}{4}$

$\boxed{y = \pm\frac{\sqrt{3}}{2}}$



$m = \frac{y}{x}$

at $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ $\quad m = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3}$

at $\left(\frac{3}{2}, -\frac{\sqrt{3}}{2}\right)$ $\quad m = -\frac{\sqrt{3}}{3}$

Point $\left(\frac{3}{2}, \frac{\sqrt{3}}{3}\right)$, Slope $\frac{\sqrt{3}}{3}$ $\Rightarrow$ Find the line

Point $\left(\frac{3}{2}, \frac{-\sqrt{3}}{3}\right)$ Slope $\frac{-\sqrt{3}}{3}$ $\Rightarrow$ Find the line find $\dfrac{d^2y}{dx^2}$ for $x\cos y = y$

$$\frac{d}{dx}\left[x\cos y\right] = \frac{d}{dx}\left[y\right]$$

$$1\cdot\cos y + x\cdot -\sin y\cdot\frac{dy}{dx} = \frac{dy}{dx}$$

$$\cos y - x\sin y\frac{dy}{dx} = \frac{dy}{dx}$$

$$\cos y = x\sin y\frac{dy}{dx} + 1\frac{dy}{dx}$$

$$\cos y = (1 + x\sin y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\cos y}{1 + x\sin y}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{\cos y}{1 + x\sin y}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-\sin y\cdot\frac{dy}{dx}\cdot(1 + x\sin y) - \cos y\left[1\cdot\sin y + x\cdot\cos y\cdot\frac{dy}{dx}\right]}{(1 + x\sin y)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{\sin 2y + y(\sin^2 y + 1)}{(1 + x\sin y)^3}}$$

try to verify this